# Quantum superposition principle and generation of ultrashort optical pulses

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## Abstract

We discuss the propagation of laser radiation through a medium of quantum prepared  $\Lambda$ -type atoms in order to enhance the insight into the physics of QSPT generator suggested in Phys. Rev. A 80, 035801 (2009). We obtain analytical results which give a qualitatively correct description of the outcoming series of ultrashort optical pulses and show that for the case of alkali vapor medium QS-PT generation may be implemented under ordinary experimental conditions.

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#### I. INTRODUCTION

Atomic coherence is a widely accepted resource in atomic and laser physics for modifying the response of coherently prepared atomic systems and thereby canceling or enhancing the light absorption/reflection near atomic resonance [1-4]. Quantum prepared medium in fact is essentially a new state of matter, named in [5] phaseonium. The basic idea behind almost all such models is to modify the emissive and absorptive profiles and make them non-reciprocal. Among most important applications of quantum prepared medium, both in theory and experimental science, one should list adiabatic population transfer [6-9], amplification and lasing without inversion [10-13], sub-recoil laser cooling [14-17], light storage and stopping [18-21], atomic spectroscopy [22-26], etc.

On the other hand, many problems in atomic physics, such as precision optical frequency metrology, excitation spectroscopy and quantum state engineering [27-30], require a train of high-energy and high-repetition optical pulses. The most convenient method to accomplish this requirement today is the acousto-optic modulator [31], but still new principles and schemes are suggested in literature. A promising approach to this end is the usage of quantum coherency created between atomic levels in both bare and dressed approximations. For example, a considerable attention has been drawn to the parametric beating of a weak

probe field in the scheme of stimulated Raman scattering [32,33]. In this paper we suggest a new step in this direction.

We consider propagation of an incident wave through a medium of  $\Lambda$ -type three-level atoms with a preliminary superposition between the low lying doublet states, a simplest construction for experimental realizations. We carry a detailed analysis of how the superposition nature of atomic states splits the incident monochromatic wave into a train of ultrashort (and high power) pulses in a three-level medium. Such a qualitatively new and dramatic effect of superposition principle has already been identified in our previous papers [34,35]. However, it has proceeded from a two-level medium and two (probe and pump) optical fields, thus somewhat disguising the inherent ability of superposition principle in this problem. Simultaneously, the technical realization of originally proposed scheme seems stringent as much as it is difficult to onset the preliminary quantum superposition between the energy levels in the optical range of Bohr frequencies. Presented in this paper scheme clarifies the main idea of [34,35] and extends its formalism into much headmost ideological and experimental conditions using  $\Lambda$ -type interaction scheme and only one travelling wave.

#### II. ADIABATIC STATES OF $\Lambda$ -TYPE ATOM

The basic setup of interaction used throughout this article is presented in Fig. 1. The lower atomic states 1 and 2 are separated in energy by  $E_{a_2} - E_{a_1} = \hbar \Delta_0$ , and are coupled with the upper state by a plane optical field of frequency  $\omega$ :

$$\overrightarrow{E} = \overrightarrow{E}_0(z,t) \exp(i\omega t - ikz) + \overrightarrow{E}_0^*(z,t) \exp(-i\omega t + ikz), \tag{1}$$

where  $k = \omega/c$  is the vacuum wave number.

The amplitude  $\overrightarrow{E}_0(z,t)$  is assumed to vary slowly compared to the rapidly oscillating exponential functions, i.e.  $\left|\partial \overrightarrow{E}_0(z,t)/\partial t\right| << \left|\omega \overrightarrow{E}_0(z,t)\right|$  and  $\left|\partial \overrightarrow{E}_0(z,t)/\partial z\right| << \left|k \overrightarrow{E}_0(z,t)\right|$ . (Later we will take it as a constant when entering the gas medium, which is the most convenient form for implication purposes). We also assume that the necessary conditions to preserve the coherency are ensured. Dephasing from the upper state is eliminated by assuming a far off-resonant interaction, while the relaxation or dephasing between ground levels 1 and 2 lasts milliseconds and is sufficient for the objectives of this paper. Thus we will

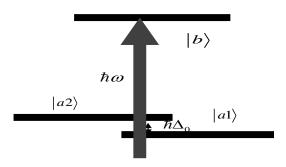


FIG. 1: Energy-level diagram for the resonant single-photon process. States  $|a1\rangle$  and  $|a2\rangle$  are of like parity whereas the upper state  $|b\rangle$  is of opposite parity. Energy distance  $\hbar\Delta_0$  between the doublet states is much smaller than the photon energy  $\hbar\omega$ .

discuss the atom-radiation coupling staying in the limits of Schrödinger equation

$$i\hbar \frac{\partial \Psi(\overrightarrow{r},t)}{\partial t} = \left(\widehat{H}_0 - \overrightarrow{d} \overrightarrow{E}\right) \Psi(\overrightarrow{r},t). \tag{2}$$

The interaction is taken in electric dipole approximation,  $\widehat{H}_0$  and  $\widehat{d}$  are free atom Hamiltonian and dipole moment operator respectively, and the atomic motion isn't included in the discussion.

We look for the solution of Eq.2 in a form

$$\Psi(\overrightarrow{r},t) = a_1(t)\psi_1(\overrightarrow{r})\exp(-iE_{a_1}t/\hbar) + a_2(t)\psi_2(\overrightarrow{r})\exp(-iE_{a_2}t/\hbar) + b(t)\varphi(\overrightarrow{r})\exp(-iE_{b_1}t/\hbar),$$
(3)

expanded over free atom doublet and excited state eigenstates  $\psi_{1,2}(\overrightarrow{r})$  and  $\varphi(\overrightarrow{r})$  respectively.  $E_{a_{1,2}}$  and  $E_b$  are corresponding energies. After substituting (2) into (3) we apply the RWA neglecting terms with rapidly oscillating factor  $\exp(\pm i(\omega + \omega_{1,2})t)$  with  $\omega_{1,2} = E_b - E_{a_{1,2}}/\hbar$ . We want to describe the phenomena under conditions where the reactive part of the atom-field coupling is predominant over the dissipative part so we proceed working in the range of resonance detunings much larger than the Doppler and homogeneous widths of optical transitions. This simultaneously gives the possibility of first order adiabatic elimination procedure, algebraically presenting the excited state amplitude through the lower lying ones:

$$b(t) = -\frac{\overrightarrow{d}_{1}^{*} \overrightarrow{E}_{0}^{*} \exp(ikz)}{\hbar(\omega - \omega_{1})} \exp(-i(\omega - \omega_{1})t)a_{1}(t) - \frac{\overrightarrow{d}_{2}^{*} \overrightarrow{E}_{0}^{*} \exp(ikz)}{\hbar(\omega - \omega_{2})} \exp(-i(\omega - \omega_{2})t)a_{2}(t).$$
(4)

Here  $\overrightarrow{d}_1$  and  $\overrightarrow{d}_2$  are a1-b and a2-b optical transition matrix elements correspondingly. For simplicity of later calculations we redefine the level 2 probability amplitude

$$A_2(t) = a_2(t) \exp(-i\Delta_0 t), \quad A_1(t) = a_1(t),$$
 (5)

and rescale the problem parameters to dimensionless ones:

$$\tau \equiv \Delta_0 t, \, \frac{\omega - \omega_1}{\Delta_0} \equiv \omega - \omega_1, \, \frac{\omega - \omega_2}{\Delta_0} \equiv \omega - \omega_2, \tag{6}$$

$$\xi_{1}(z,\tau) = \frac{\overrightarrow{d}_{1}\overrightarrow{E}_{0}(z,\tau)}{\hbar\Delta_{0}}, \quad \xi_{2}(z,\tau) = \frac{\overrightarrow{d}_{2}\overrightarrow{E}_{0}(z,\tau)}{\hbar\Delta_{0}}.$$
 (7)

The doublet amplitudes now will take a relatively simple form

$$i\frac{dA_{1}(\tau)}{d\tau} - \frac{\xi_{1}^{*}(z,\tau)\xi_{1}(z,\tau)}{\omega - \omega_{1}}A_{1}(\tau) = \frac{\xi_{1}(z,\tau)\xi_{2}^{*}(z,\tau)}{\omega - \omega_{2}}A_{2}(\tau),$$

$$i\frac{dA_{2}(\tau)}{d\tau} - A_{2}(\tau) - \frac{\xi_{2}^{*}(z,\tau)\xi_{2}(z,\tau)}{\omega - \omega_{2}}A_{2}(\tau) = \frac{\xi_{2}(z,\tau)\xi_{1}^{*}(z,\tau)}{\omega - \omega_{1}}A_{1}(\tau).$$
(8)

They take up stationary solutions

$$A_1(\tau) = A_1(0) \exp(-i\lambda_{1,2}\tau), \quad A_2(\tau) = A_2(0) \exp(-i\lambda_{1,2}\tau)$$
 (9)

with characteristic values

$$\lambda_1 = \frac{1}{2} \left( p + \sqrt{p^2 - 4q} \right), \quad \lambda_2 = \frac{1}{2} \left( p - \sqrt{p^2 - 4q} \right),$$
 (10)

where

$$p = 1 + \frac{\xi_1^* \xi_1}{\omega - \omega_1} + \frac{\xi_2^* \xi_2}{\omega - \omega_2}, \quad q = \frac{\xi_1^* \xi_1}{\omega - \omega_1}.$$
 (11)

These solutions should be normalized so that  $|A_1(0)|^2 + |A_2(0)|^2 = 1$ . Thus we obtained adiabatic solutions, mapping the superposition of three bare states into a dressed state. The solutions corresponding to the characteristic values  $\lambda_1$  and  $\lambda_2$  have the form

$$\Psi(t,\lambda_{1,2}) = \begin{pmatrix} A_1(0,\lambda_{1,2})\psi_1(\overrightarrow{r}) \exp(-iE_{a_1}t/\hbar) + A_2(0,\lambda_{1,2})\psi_2(\overrightarrow{r}) \exp(-iE_{a_2}t/\hbar + i\tau) \\ +b(0,\lambda_{1,2})\varphi(\overrightarrow{r}) \exp(-iE_bt/\hbar) \end{pmatrix} \exp(-iD_{a_1}t/\hbar) \exp(-iD_{a_2}t/\hbar + i\tau)$$
(12)

where

$$A_{1}(0,\lambda_{1,2}) = \frac{\xi_{2}^{*}\xi_{1}}{(\omega - \omega_{2})(\lambda_{1,2} - q)\sqrt{1 + \xi_{1}^{*}\xi_{1}\xi_{2}^{*}\xi_{2}/(\omega - \omega_{2})^{2}(\lambda_{1,2} - q)^{2}}},$$

$$A_{2}(0,\lambda_{1,2}) = \frac{1}{\sqrt{1 + \xi_{1}^{*}\xi_{1}\xi_{2}^{*}\xi_{2}/(\omega - \omega_{2})^{2}(\lambda_{1,2} - q)^{2}}}.$$
(13)

Both excited state amplitudes  $b(0, \lambda_1)$  and  $b(0, \lambda_2)$  are determined from Eq.(4).

# III. LIGHT PROPAGATION IN QUANTUM PREPARED MEDIUM. SIMPLE APPROXIMATE ANALYTIC SOLUTION

Now we proceed to the discussion of the field propagation in the medium using wave equation

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \overrightarrow{E} = \frac{4\pi\rho}{c^2} \frac{\partial^2}{\partial t^2} \langle \Psi(t) | \widehat{\overrightarrow{d}} | \Psi(t) \rangle, \qquad (14)$$

where the quantity  $\rho \partial \langle \Psi(t) | \widehat{\overrightarrow{d}} | \Psi(t) \rangle / \partial t$  stands for the density of the bias electric current in dielectric gas medium,  $\rho$  is the number of atoms per unit volume and the brackets mean quantum mechanical plus thermal state averaging. The form above in fact assumes that the gas density is low enough and the temperature is high enough for quantum mechanical collective effects to be irrelevant. We make the ansatz

$$\langle \Psi(t) | \widehat{\overrightarrow{d}} | \Psi(t) \rangle = \frac{1}{1 + \exp\left(-\frac{E_{a_2} - E_{a_1}}{k_B T}\right)} \langle \alpha \Psi(t, \lambda_1) + \beta \Psi(t, \lambda_2) | \widehat{\overrightarrow{d}} | \alpha \Psi(t, \lambda_1) + \beta \Psi(t, \lambda_2) \rangle + \frac{1}{1 + \exp\left(\frac{E_{a_2} - E_{a_1}}{k_B T}\right)} \langle \overline{\alpha} \Psi(t, \lambda_1) + \overline{\beta} \Psi(t, \lambda_2) | \widehat{\overrightarrow{d}} | \overline{\alpha} \Psi(t, \lambda_1) + \overline{\beta} \Psi(t, \lambda_2) \rangle,$$

$$(15)$$

which assumes independent formation of superposition states from the thermally populated levels 1 and 2. The coefficients  $(\alpha, \beta)$  and  $(\overline{\alpha}, \overline{\beta})$  are the probability amplitudes corresponding to each channel.

Standard transformations in frame of slowly varying amplitudes approximation bring us to a reduced form of wave equation (14), which for a better insight of the field dynamics we write in the following symbolic form:

$$\left(\frac{\partial}{\partial z} - \frac{1}{c}\frac{\partial}{\partial t}\right)\eta\left(z,\tau\right) = i\left(F\left(|\eta|^2\right) + G_C\left(|\eta|^2\right)\cos((\lambda_2 - \lambda_1)\tau) + G_S\left(|\eta|^2\right)\sin((\lambda_2 - \lambda_1)\tau)\right)\eta\left(z,\tau\right).$$
(16)

The wave amplitude  $\eta\left(z,\tau\right)$  is defined in a dimensionless, symmetric with respect to both 1-3 and 2-3 transitions form

$$\eta(z,\tau) = \frac{2d_1 d_2 E_0(z,\tau)}{(d_1 + d_2)\hbar \Delta_0}.$$
(17)

The right-hand side expression in the brackets conditions the response of medium to the coupling field and the nature of the field evolution. Note that there is nothing surprising in the fact that the response of medium in superposition state is time dependent, as is seen

from Eq.(16), which leads to phase modulation and relevant phenomena. The "hidden" from the first glance and crucial point here is the existence of an imaginary part in the response function for this non-dissipative medium under consideration. Periodic in time imaginary part acknowledges the quantum superposition nature of atomic state prior to the atom-field interaction and results in sequential amplification and suppressing of the field intensity during the propagation. And this is the reason (or origin) of the birth of series of short or ultrashort optical pulses, which we term as quantum superposition pulse train (QS-PT) generator.

We will not discuss in this paper the questions connected with phase modulation and proceed to a solution of the problem in terms of the field intensity  $|\eta(z,\tau)|^2$ . The wave equation for it follows from Eq.(16):

$$\frac{\partial}{\partial \widetilde{\zeta}} \left| \eta \left( \widetilde{\zeta}, \widetilde{\tau} \right) \right|^{2} = i \frac{2\pi \rho \omega d_{1} d_{2}}{\hbar \Delta_{0}^{2}} \left( \frac{1}{\omega - \omega_{2}} - \frac{1}{\omega - \omega_{1}} \right) \times \left( \left( \frac{\alpha^{*} \beta}{1 + \exp\left(-\frac{\hbar \Delta_{0}}{k_{B}T}\right)} + \frac{\overline{\alpha^{*} \beta}}{1 + \exp\left(\frac{\hbar \Delta_{0}}{k_{B}T}\right)} \right) \exp\left(-i\left(\lambda_{2} - \lambda_{1}\right) \left(\widetilde{\zeta} + \widetilde{\tau}\right)\right) - \left( \frac{\alpha \beta^{*}}{1 + \exp\left(-\frac{\hbar \Delta_{0}}{k_{B}T}\right)} + \frac{\overline{\alpha \beta}^{*}}{1 + \exp\left(\frac{\hbar \Delta_{0}}{k_{B}T}\right)} \right) \exp\left(i\left(\lambda_{2} - \lambda_{1}\right) \left(\widetilde{\zeta} + \widetilde{\tau}\right)\right) \right) \times (A_{2}(0, \lambda_{1}) A_{1}(0, \lambda_{2}) - A_{1}(0, \lambda_{1}) A_{2}(0, \lambda_{2})) \left| \eta \left(\widetilde{\zeta}, \widetilde{\tau}\right) \right|^{2}, \tag{18}$$

with  $\tilde{\zeta} = \Delta_0 z/c$ ,  $\tilde{\tau} = \Delta_0 (t-z/c)$ . Eq.(18) is the main result of this paper and the basis of later discussion. We should note that in general it is a nonlinear equation and nonlinearity enters through the intensity-dependence of characteristic values  $\lambda_1$  and  $\lambda_2$ . As we can see they appear in the equation in two forms:  $A_2(0,\lambda_1)A_1(0,\lambda_2) - A_1(0,\lambda_1)A_2(0,\lambda_2)$  and  $\lambda_2 - \lambda_1$ . Numerical calculations, see Fig.2, show that the first one of them, denoted  $[A_2,A_1]$  with high accuracy may be treated as not depending on wave intensity. The difference  $\lambda_2 - \lambda_1$  may also be viewed as as intensity independent, but only when  $\eta(z,\tau) << 1$  and equals to unity  $(\lambda_2 - \lambda_1 = 1)$ . For this intensity range the problem becomes to a great extend exactly soluble and has the following general solution form:

tend exactly soluble and has the following general solution form:
$$|\eta(z,t)|^2 = |\eta(0,t)|^2 \exp \begin{pmatrix} \frac{2\pi\rho\omega d_1 d_2}{\hbar\Delta_0^2} \left(\frac{1}{\omega-\omega_2} - \frac{1}{\omega-\omega_1}\right) \times \\ (A_2(0,\lambda_1) A_1(0,\lambda_2) - A_1(0,\lambda_1) A_2(0,\lambda_2)) \times \\ \left(\frac{\alpha^*\beta}{1+\exp\left(-\frac{\hbar\Delta_0}{k_BT}\right)} + \frac{\overline{\alpha^*\beta}}{1+\exp\left(\frac{\hbar\Delta_0}{k_BT}\right)}\right) \times \\ \frac{1-\exp(-i(\lambda_2-\lambda_1)(\Delta_0z/c))}{\lambda_2-\lambda_1} \exp\left(-i(\lambda_2-\lambda_1)\Delta_0t)\right) + c.c. \end{pmatrix}, (19)$$

where the characteristic values  $\lambda_1$  and  $\lambda_2$  are determined for the given initial wave-front.

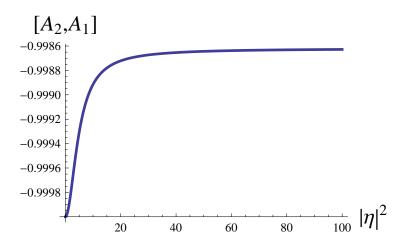


FIG. 2: Functional form  $[A_2,A_1]$  (see the text) as function of scaled field intensity  $\eta$ . Atomic transition frequencies  $\omega_1=287351\Delta_0$  and  $\omega_2=287350\Delta_0$  imply sodium atom's spectrum (see paragraph III of the main text), and input laser frequency  $\omega=287360\Delta_0$  is detuned from these an order of magnitude larger than the doublet splitting  $\Delta_0=\omega_1-\omega_2$ .

This formula shows all the immense advantage of the scheme under consideration: the output intensity is modulated at the frequency  $\lambda_2 - \lambda_1$ , which is the distance between the ground doublet levels. This is a remarkable and a prominent performance index of the presented scheme, since a very wide range of high stability repetition rates can be attained for the pulse trains by simply choosing different atomic species with doublet (or more) ground level structures.

The second prominent advantage of the presented scheme is the presence of a prefactor  $1/\Delta_0^2$  instead of  $\frac{\Omega/\Delta}{(1+(\Omega/\Delta)^2)^2}\frac{1}{\Delta^2}$  in the original version [34], where  $\Delta$  is detuning of the intense pump field relative to the two-level optical transition. To neglect the spontaneous emission, one usually has to work in the range where detunings are  $\Delta > 10^{11}Hz$ , which is at least one order of magnitude larger than  $\Delta_0$  for alkaline metal gases. At the same time in old version we are limited with  $\Omega/\Delta = 0.1$  upper value in the additional coefficient. So the present scheme possesses much more freedom for such parameter values as gas density, and perhaps can be implemented even on weak optical transitions.

Another preferable side of  $\Lambda$ -type interaction scheme is the acquisition of superposition between the closely posed ground states, as it is not a difficult task and may be implemented by means of magnetic fields as well. In case of a preparative radiation field, one has  $\alpha^* = \alpha$ ,  $\beta^* = -\beta$ ,  $\overline{\alpha}^* = -\overline{\alpha}$  and  $\overline{\beta}^* = -\overline{\beta}$ . The expression (19) becomes somewhat simplified and the relative intensity  $|\eta(z,t)|^2 / |\eta(0,t)|^2$  (RI) takes the ultimate form

Relative intensity= exp 
$$\left( \frac{\frac{4\pi i \rho \omega d_1 d_2}{\hbar \Delta_0^2} \left( \frac{1}{\omega - \omega_2} - \frac{1}{\omega - \omega_1} \right) \left( \frac{\alpha \beta}{1 + \exp\left(-\frac{\hbar \Delta_0}{k_B T}\right)} + \frac{\overline{\alpha \beta}}{1 + \exp\left(\frac{\hbar \Delta_0}{k_B T}\right)} \right) \times \frac{A_2(0, \lambda_1) A_1(0, \lambda_2) - A_1(0, \lambda_1) A_2(0, \lambda_2)}{\lambda_2 - \lambda_1} \left( \sin\left((\lambda_2 - \lambda_1) \left(\tau - \Delta_0 z/c\right)\right) - \sin\left((\lambda_2 - \lambda_1) \tau\right) \right) \right)$$

$$(20)$$

Expression in the exponent periodically oscillates as a function of time and coordinate around the zero value. Therefore, in principle two types of field envelope transformations are possible: propagation with a periodic amplification and propagation with a periodic suppression. The answer to the question which process will be realized for a given superposition state is determined through the values of  $\omega - \omega_1$  and  $\omega - \omega_2$  differences.

### IV. NUMERICAL ANALYSIS OF PULSE TRAIN FORMATION

For quantitative analysis we discuss the case of  $3S_{1/2} - 3P_{1/2}$  transition of  $^{23}Na$ . The hyperfine levels F = 1 and F = 2 of ground level  $3S_{1/2}$  constitute the doublet structure discussed. As the hyperfine splitting in the excited state  $3P_{1/2}$  is orders of magnitude smaller than the 1771.6 MHz splitting of  $3P_{1/2}$ , we discuss the former as a collection of degenerate sublevels of same energy. We are also allowed to neglect all thermal transitions between ground levels F = 1 and F = 2 during the interaction.

The incident wave is taken circularly polarized and this polarization is preserved during propagation through the medium. Transition matrix elements now should be calculated between the magnetic sublevels, and this should include selection rules of the total moment F and its projection M on the quantization axis aligned with the propagation direction. This, as usually, is realized using formulas (see, for example [36])

$$\langle F, M - 1 | \widehat{d}_{-} | F, M \rangle = \sqrt{\frac{(F - M + 1)(F + M)}{F(F + 1)(2F + 1)}} \langle F | \widehat{d} | F \rangle,$$

$$\langle F, M - 1 | \widehat{d}_{-} | F - 1, M \rangle = \sqrt{\frac{(F - M + 1)(F - M)}{F(2F - 1)(2F + 1)}} \langle F | \widehat{d} | F - 1 \rangle,$$

$$\langle F - 1, M - 1 | \widehat{d}_{-} | F, M \rangle = \sqrt{\frac{(F + M + 1)(F + M)}{F(2F - 1)(2F + 1)}} \langle F - 1 | \widehat{d} | F \rangle,$$

$$\langle F', M' | \widehat{d}_{+} | F, M \rangle = (\langle F', M | \widehat{d}_{-} | F', M \rangle)^{*},$$

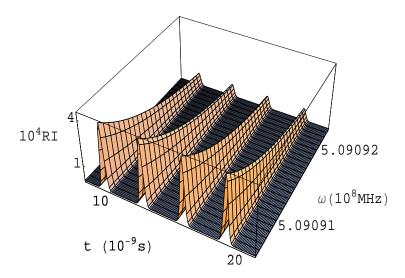


FIG. 3: Splitting of monochromatic incident wave intensity into a series of ultrashort pulses in a quantum prepared three-level atomic medium. Here  $z\Delta_0/c=\pi$ ,  $\rho=4\times 10^{12}cm^{-3}$ ,  $T=10^{-6}K$ ,  $\alpha=\overline{\beta}=0.5,\ \beta=\overline{\alpha}=i\sqrt{0.75}$ . All the other parameters are the same as in Fig.2.

separating a projection-dependent prefactor from the reduced matrix element  $\langle F' | \hat{d} | F \rangle$ . Here  $\hat{d}_{\pm} = \hat{d}_x \pm i\hat{d}_y$  are circular components of dipole moment operator. Finally it was convenient to use the formulae

$$\langle F' | \hat{d} | F \rangle = \sqrt{\frac{3\hbar e^2}{2m\omega_{FF}} (2F+1)f(F \to F')},$$
 (21)

introducing the oscillator strength  $f(F \to F')$  instead of reduced matrix element  $\langle F' | \hat{d} | F \rangle$ .

The expression for the light field amplitude is of Eq.(19) form. The depth of modulation increases with atom density and thereby decreases the duration of each individual pulse in the sequence. Pulse duration may be significantly controlled using resonance detuning too. Fig.3 graphically illustrates the situation, plotting the envelope of the field intensity as a function of time for various frequency detunings. Though the parameters are chosen for illustrative purposes, they are realistic for ordinary laboratory conditions. As might have been expected, pulse repetition rate is irrelevant to resonance detuning and mimics the doublet splitting frequency. As to pulse duration, is around hundred picoseconds for the

chosen frequency region and concomitant with power amplification. One will have the same picture for spatial evolution too.

# V. CONCLUSIONS AND OUTLOOK

We showed that the gas of quantum prepared  $\Lambda$ -type atoms is very efficient in splitting of monochromatic wave into a sequence of ultrashort pulses, enlarging the spectrum of pulse generation mechanisms suggested or implemented so far. We emphasize that the repetition rate of ultrashort pulses does not depend neither on the light intensity nor its detuning from the atomic resonance. The sole quantity determining the repetition rate is the frequency separation between the ground doublet levels between which the superposition has been preliminarily established. Duration of each pulse, vice versa, is rigorously dependent on coupling parameters and effectively controllable. These regularities accompanied by mild conditions for the experimental realization presents the QS-PT generator as a viable alternative to more conventional methods.

Finally we would like to note, that since the QS-PT generator is inevitably grounded on the quantum superposition principle, the studied phenomenon of abrupt shape-reformation of the classical electromagnetic field envelope may be regarded as an example of quantum macroscopic effect.

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